International Journal of Theoretical Physics, Vol. 2, No. 3 (1969), pp. 247–254.

Unified Field Theory of Quarks and Electrons

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Received: 4 *April* 1969

Abstract

A unified gauge theory of quarks and electrons in which electromagnetic and strong interactions are both transmitted by the same vector field is formulated. It is argued that the theory is finite and that it is in agreement with observed electromagnetic and nuclear phenomena.

1. *Introduction*

For the purpose of describing accurately a large class of low-energy electromagnetic phenomena, conventional quantum electrodynamics has been an extremely successful theory. However, it is widely known that it contains a serious flaw—at high energies it is beset with divergence difficulties. The fact that while its divergences occur at high energies, its confirmed successes occur at low energies, suggests that quantum electrodynamics may in reality be the low-energy limit of a 'correct' theory, which theory may be assumed to be finite. This possibility is examined here by reference to a sequence of specific models.

2. *Quantum Electrodynamics*

The guiding principle in the following is the assumption that the correct theory is finite *prior to renormalization*. Conventional quantum electrodynamics, as characterized by the interaction Lagrangian

$$\mathscr{L} = e_0 \bar{\psi}\gamma_\mu \psi A_\mu \tag{2.1}$$

has recently been shown to satisfy this criterion in the following sense. The unrenormalized Schwinger-Dyson equations for the electron propagator S and the 3-point vertex function Γ have been shown by Johnson, *et al.* (1964; hereafter referred to as JBW) to have finite solutions provided the bare mass of the electron vanishes, and provided also that the photon propagator D is finite. This last requirement is satisfied for particular values of the bare coupling α_0 ($\equiv e_0{}^2/4\pi^2$). For, Gell-Mann & Low (1954) and Johnson, *et* *al.* (1967; hereafter referred to as JWB) have obtained eigenvalue equations for which they have shown, if α_0 is a root, then D is finite. The JWB eigenvalue equation, which must have the same roots as the Gell-Mann & Low equation and on which we somewhat arbitrarily choose to base the following remarks, may be written in the form

$$\alpha_0 + \alpha_0 f(\alpha_0) = 0 \tag{2.2}$$

where $f(\alpha_0)$ is the ratio

$$f(\alpha_0) = \frac{(\text{4th + higher-order JWB contributions to } Z_3^{-1})}{(\text{2nd-order JWB contribution to } Z_3^{-1})} \tag{2.3}$$

Here $Z_3 = e^2/e_0{}^2$ and, as stated, all contributions are to be calculated in the special JWB approximation scheme [i.e. iteration on $\Gamma_\mu(p,p)$]. The ratio (2.3) is finite because, as was shown by JWB, although each contribution

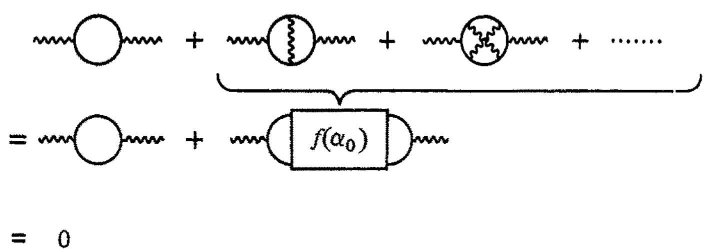

Figure 1—Equation (2.2) in diagrammatic form. By loosely equating the sum of diagrams to zero we mean here that the sum is not to equal infinity. The precise statement is given by equations (2.2) and (2.3). Note that to calculate Z_3 each diagram must be doubly differentiated in the manner described by Johnson, *et al.*, 1967.

to Z_3^{-1} diverges, they do so in the same say (i.e. as a *single* power of the logarithm of a cutoff). Equation (2.2) may be represented diagrammatically as shown in Fig. 1.

The sole remaining question is whether or not equation (2.2) has an acceptable root for α_0. Since the function $f(\alpha_0)$ is not yet amenable to exact evaluation, a definitive answer to this fundamental question cannot presently be given. We are therefore forced to resort to the following semi-quantitative argument. We begin by rewriting equation (2.2) in the form

$$f(\alpha_0) = -1 \tag{2.4}$$

We then note that in the JBW calculation the empirical condition that, for presently attainable laboratory momenta p the electron propagator is proportional to $(\gamma p + m)^{-1}$, where m is the observed electron mass, is satisfied if (and almost certainly only if)

$$\alpha_0 \ll 1 \tag{2.5}$$

This, together with the smallness of renormalized (i.e. observed) charge and the definition (2.3), suggest that $f(\alpha_0)$ can be expanded in a (convergent) power series of the form

$$f(\alpha_0) = O(\alpha_0) + O(\alpha_0{}^2) + O(\alpha_0{}^3) + \cdots \tag{2.6}$$

The crux of the matter is then whether or not equation (2.4) and requirement (2.5) are compatible. It is suggested by (2.6) that they are not, but it is of course true that until the complete series is evaluated no definite conclusion can be drawn. The incompatibility of (2.4) and (2.5) is, however, even more likely than that suggested above. This is because explicit calculation has indicated that the $O(\alpha_0)$ term in (2.6) is positive (Yock, 1968. Earlier calculations of the $O(\alpha_0)$ term are referred to in this paper). This would appear to preclude any reasonable possibility for a small positive root to equation (2.4). We therefore discard in this paper theory (2.1) as probably being either inconsistent with observed data or divergent.

Note that, should (2.4) and (2.5) be compatible, then the JBW conclusion that $S(p) \simeq (\gamma p + m)^{-1}$ for $p \ll m \exp(2/3\alpha_0)$ would have to be re-examined, since their calculation assumes that high-order terms are negligible.

3. *Conventional Gauge Theory of Electromagnetic and Nuclear Interactions*

Besides probably being divergent at high energies, theory (2.1) is also lacking in that it neglects strong interactions. To test the possibility that these two shortcomings may be related, we now examine a combined model of strong and electromagnetic interactions. The interaction Lagrangian is

$$\mathscr{L} = g_0 j_\mu{}^p B_\mu + e_0 j_\mu{}^p A_\mu + e_0 j_\mu{}^e A_\mu \tag{3.1}$$

where

$$j_\mu{}^p = \bar{p}\gamma_\mu p \qquad \text{and} \qquad j_\mu{}^e = \bar{\psi}\gamma_\mu \psi \tag{3.2}$$

Here p denotes a spinor proton field and B_μ a neutral vector field. We note that gauge theories fundamentally similar to (3.1) have been considered previously by many authors (Fermi & Yang, 1949; Lee & Yang, 1955; Sakurai, 1960; Ne'eman, 1961; Schwinger, 1965). The Dirac field equations imply that the currents $j_\mu{}^p$ and $j_\mu{}^e$ are separately conserved. The coupling g_0 is assumed to be strong, whereas e_0 is assumed to be relatively weak. Thus, denoting $g_0{}^2/4\pi^2$ by β_0, we have

$$\beta_0 \gg \alpha_0 \ll 1 \tag{3.3}$$

More precisely, we envisage that β_0 may be $\gtrsim 1$.† Theory (3.1) is to be considered here as at best a simple model of the observed strong and electromagnetic interactions.

† We refer of course to the natural system of units in which $\hbar = c = 1$.

Provided the bare masses of the fields A_μ, B_μ, ψ and p all vanish, only infinities associated with the A_μ and B_μ propagators can occur in theory (3.1). This follows, via the work of JBW, from the freedom of choice of gauge in theory (3.1). The condition that the B_μ propagator be finite evidently takes the form

$$\beta_0 + \beta_0 f(\beta_0) \simeq 0 \tag{3.4}$$

where f is the function defined by equation (2.3), but here evaluated at β_0.† Equation (3.4), which is depicted in Fig. 2, follows as a consequence of the

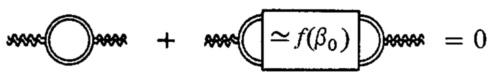

Figure 2—Equation (3.4) in diagrammatic form. The double wavy line represents the β_μ field, and the non-wavy double line the proton field.

fact that in analysing the B_μ propagator electromagnetic effects must necessarily be small if (3.3) is true. The condition that the A_μ propagator be finite similarly takes the form (see Fig. 3)

$$\alpha_0 + \alpha_0 + \alpha_0 f(\beta_0) \simeq 0 \tag{3.5}$$

assuming once again equation (3.3).

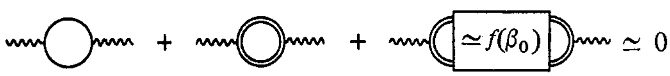

Figure 3—Equation (3.5) in diagrammatic form.

Equations (3.4) and (3.5) are manifestly inconsistent. We therefore discard theory (3.1) as being divergent.

4. *Unified Gauge Theory*

The previous theory failed in its inability to provide for simultaneously finite A_μ and B_μ propagators. The following 'unified' theory, which involves a single vector field only, is designed specifically to circumvent this difficulty. The interaction Lagrangian is

$$\mathscr{L} = g_0 j_\mu{}^p A_\mu + e_0 j_\mu{}^e A_\mu \tag{4.1}$$

where, as before, the proton and electron currents are $\bar{p}\gamma_\mu p$ and $\bar{\psi}\gamma_\mu\psi$ respectively. They are separately conserved. The couplings are assumed again to satisfy equation (3.3).

Provided the bare mass of the 'electromagnetic' field A_μ vanishes, theory

† For values of β_0 exceeding the radius of convergence of the power series in equation (2.3), $f(\beta_0)$ must be defined specifically by the summation shown in Figure 1.

(4.1) still permits an arbitrary choice of gauge. Hence the theory is finite if a charge eigenvalue equation of the form

$$\beta_0 + \beta_0 f(\beta_0) \simeq 0 \tag{4.2}$$

is satisfied. This follows from equation (3.3). The mild assumption that equation (4.2) has for β_0 a positive root satisfying (3.3) (which root is presumably $\gtrsim 1$) completes our demonstration that theory (4.1) is finite.

Having constructed an apparently finite theory, it must now be seen if it yields dynamical predictions in agreement with observed phenomena. Clearly, since $\beta_0 \gg \alpha_0$, theory (4.1) predicts protonic interactions to be universally stronger than electronic interactions.† But this is inconsistent with, for example, the observed equality (via Rutherford and Møller scattering) of the long-range electron-electron and proton-proton forces. Hence we discard theory (4.1) as, although probably finite, nevertheless seriously inconsistent with observed data.

5. *Unified Gauge Theory of Quarks and Electrons*

The failure of the previous unified gauge theory is very suggestive. Referring to g_0 as 'hadronic' charge we have seen that the finiteness of the 'photon' propagator D necessitates the existence of fields carrying large hadronic charge. On the other hand, the observed nucleons cannot be directly associated with these fields because such nucleons would not obey Coulomb's law. The simplest way out of this dilemma is to assume that the observed nucleons are aggregates of fundamental spinor particles in which the net hadronic charge cancels. Bearing this in mind, and referring to the basic hadronic charge-carrying particles as 'quarks',‡ we are thus led to consider the following two-quark model:

$$\mathscr{L} = \{e_0 j_\mu{}^e + (2g_0 + e_0) j_\mu{}^Q + g_0 j_\mu{}^q\} A_\mu \tag{5.1}$$

where

$$\begin{aligned} j_\mu{}^e &= \bar{\psi}\gamma_\mu \psi \\ j_\mu{}^Q &= \bar{Q}\gamma_\mu Q \\ j_\mu{}^q &= \bar{q}\gamma_\mu q \end{aligned} \tag{5.2}$$

The remainder of this paper is devoted to discussion of the theory defined by the interaction (5.1). We first note that, provided the bare masses of the fields ψ, A_μ, q and Q all vanish, then the theory is finite for a certain value of g_0 once the value of e_0 has been assigned.§ In what follows we assume, in accordance with the discussion in Section 2, that this pre-assigned value of e_0 is small. Then g_0 must be $\gtrsim 1$.

† The author thanks Professors B. T. Feld and K. A. Johnson for stressing this point.

‡ This name is taken from a theory of Gell-Mann (1964) in which it is also assumed that the observed nucleons are not fundamental particles.

§ Strictly speaking this has been verified only through each term in a power series expansion in the coupling. Our justification for relying on this procedure rests entirely on the results it yields.

Associated with the three basic fermion fields ψ, Q and q we may anticipate particles of (unrenormalized) 'charges' e_0, $2g_0 + e_0$ and g_0 respectively. Corresponding to these will be the antiparticles with charges $-e_0$, $-2g_0 - e_0$ and $-g_0$. Since g_0 must necessarily be $\gtrsim 1$ there will be strong attraction between quarks and antiquarks. Hence it is quite plausible, and this is a basic postulate, that *all readily observable states are hadronically neutral.* This suggests the following hypothetical particle classification for model (5.1):

$$
\begin{aligned}
e &= \psi \\
\pi &= \bar{Q}Q \qquad (l = 0 \text{ singlet state}) \\
\rho &= \bar{Q}Q \qquad (l = 0 \text{ triplet state}) \\
\eta &= \bar{q}q \qquad (l = 0 \text{ singlet state}) \\
\phi &= \bar{q}q \qquad (l = 0 \text{ triplet state}) \\
p &= Q\bar{q}\bar{q}
\end{aligned}
$$

and

$$\gamma = A_\mu \tag{5.3}$$

With this classification the stability of the proton is absolute, resulting as it does from the exact conservation of $j_\mu{}^Q$ and $j_\mu{}^q$. Similarly the electron is stable. On the other hand, all the meson states (except γ) are unstable.

As is indicated in equation (5.3) the spins and parities of the bound quark states may be determined as if they were entirely without orbital angular momentum. This leads to J^P assignments of 0^- for the π and η states, and 1^- for the ρ and ϕ states. It also leads to a unique prediction of $J = \frac{1}{2}$ for the proton state, since antisymmetry of the $\bar{q}\bar{q}$ wave function requires that this pair be in the singlet spin state.

Our next consideration concerns the masses of the states listed in equation (5.3). Here our first consideration must be to the masses of the quarks themselves. As is well known (Maris, *et al.*, 1965; Arnowitt and Deser, 1965) in theories like (5.1) with vanishing bare mass, the observed fermion masses are theoretically indeterminate. We shall assume here that they satisfy

$$m_e \ll m_q = m_Q \tag{5.4}$$

The first of these assumptions is based on observation, the second on simplicity. Returning now to the bound quark states, it is evident that, because of the considerably greater binding involved, the $\bar{Q}Q$ states will be considerably lighter than the $\bar{q}q$ states. This is consistent with the wide disparities observed amongst meson masses. For similar reasons we may expect $\bar{Q}Q$ states to interact considerably more strongly than $\bar{q}q$ states. This is likewise consistent with the fact that in nature it is the lightest mesons which appear to interact most strongly.

Further details of the particle interactions may be deduced from a knowledge of the form of the 'photon' propagator D. Its radiation gauge projection satisfies (Schwinger, 1962) a spectral representation of the form

$$D(q^2) = \frac{Z_3}{q^2} + \int_0^\infty \frac{\sigma(x^2)\,dx^2}{q^2 + x^2} \tag{5.5}$$

where

$$1 = Z_3 + \int_0^\infty \sigma(x^2)\,dx^2 \tag{5.6}$$

In a free (i.e. non-interacting) theory $Z_3 = 1$ and $\sigma(x^2) = 0$. When interactions are switched on $\sigma(x^2)$ receives positive contributions from pair (etc.) states and Z_3 is accordingly depressed from unity as required by the fundamental commutator relation (5.6). Clearly $\sigma(x^2)$ must be dominated by strongly interacting intermediate states. In a crude single delta function approximation we would have

$$D(q^2) \simeq \frac{Z_3}{q^2} + \frac{1 - Z_3}{q^2 + m\rho^2} \tag{5.7}$$

equation (5.7) being an expression of 'rho-dominance' in theory (5.1). At low energies ($q^2 \ll m_\rho^2$) single A_μ exchange is described by the usual (unrenormalized) photon propagation function Z_3/q^2. Accordingly the long-range ee, e^+p and pp forces are identical and equal to $Z_3 {e_0}^2/4\pi r^2 = e^2/4\pi r^2$, so that, as it was designed to do, theory (5.1) satisfies correctly Coulomb's law. At high energies an electron still interacts via single A_μ exchange, since, as we saw in Section 2, e_0 is $\ll 1$. However, two adjacent protons (or pions, etc.) undergo multiple particle exchange, so that here the constituent hadronic changes are felt and the interaction is strong.

In conclusion we remark that, with our previous assumptions, the magnetc moment of the proton state would be expected to be $\simeq g_0/m_Q$. With $g_0 \gg e$ and $m_Q \gg m_p$ this ratio could be of order e/m_p.

Acknowledgements and Outlook

The author is grateful for encouragement received from the late Professor Gunnar Källén during the early stages of this work, and also for being fortunate enough to have been able to attend the lectures of Professors K. A. Johnson and J. Schwinger at the 1964 Brandeis Summer Institute in Theoretical Physics. A definitive test of the preceding theory is obtained by asking whether or not it can be generalized in such a way that a clear correspondence is obtained between the particles actually observed in nature and the 'hadronically neutral' states of the generalized theory. One possibility, involving six quarks with charges g_0, $2g_0$, $3g_0$, $4g_0$ and $g_0 + e_0$ and $4g_0 + e_0$ respectively, appears as though it may be satisfactory in this respect. This six-quark theory, in terms of which the weak interactions take on a particularly simple and elegant form, is presently being studied in some detail. A second and even more definitive test of the theory concerns its predicted existence of highly charged quarks. Their would-be manifestations in cosmic ray interactions, especially those which are indicative of a multi-centred (or repeated) interaction mechanism, are also being studied.

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